ETALE COHOMOLOGY SYLLABUS

Lecture 1

Weil conjectures and Etale maps: In this lecture, we will aim to introduce the Weil conjectures to motivate etale cohomology and give a broad-strokes idea of how etale cohomology is used to attack these conjectures. We will then give a succinct introduction to etale maps. Time-permitting, we will introduce Grothendieck sites.

Lecture 2

Sites and Sheaves: We will continue to discuss sites, with a focus on the etale, fppf, and fpqc sites. Then, we will move onto discussing sheaves on the etale site. This will likely require at least two sessions as we will have to discuss descent and sketch a proof of faithfully flat descent. We might bleed into the next lecture a bit, in order to do descent justice.

Lecture 3

More on Sheaves: We will finish up our discussion of sheaves on the etale site, which will include some discussion of operations on sheaves like stalks/pushforward/pullback and sheafification (likely only on the etale site as that is much simpler). We will also discuss derived categories and 6-functor formalisms (time-permitting).

Lecture 4

Cohomology I:. We will begin talking about sheaf cohomology on the etale site. We will first show that the category of abelian sheaves on the etale site has enough injectives, which will allow us to define/compute sheaf cohomology as we usually do. We will also show that the etale cohomology of a point $\mathrm{Spec} k$ is the same as the Galois cohomology of $\mathrm{Gal}(k^{\mathrm{sep}}/k)$. We will also introduce the Cech complex, and the Cech-to-Derived spectral sequence for computing cohomology (for etale cohomology, Cech complexes are not a very practical way to compute cohomology, as we don't have a handy supply of acyclic covers as we did for Zariski sheaves).

Lecture 5

Cohomology II:. We will mostly focus on the etale cohomology of curves which will include a discussion of etale torsors, and Brauer groups. A complete discussion of the etale cohomology of curves will require at least 1.5 lectures, but we will see how far we get here and continue in the next lecture.

Lecture 6

Cohomology III:. We will try to finish up the discussion of the etale cohomology of curves in this lecture, and discuss compactly supported cohomology and the Gysin sequence.

Lecture 7

Various Base Change Theorems: In this lecture, we will begin discussing smooth and proper base change. There will be various lemmas leading up to these theorems which will likely take up the entire lecture (if not more). In fact, I think we will need more technology to prove smooth base change.

Lecture 8

Fundamental Groups and Some Topology: In this talk, we will introduce etale fundamental groups and discuss some of their properties. We will also discuss Kunneth and cycle class maps (it's not clear how important proofs are for things like Kunneth).

Lecture 9

More Topology: We will continue with Kunneth, cycle class maps, and talk about Chern classes and Poincare duality. We will state the Lefschetz Fixed Point Theorem and work towards a proof (time-permitting).

Lecture 10 to n

Even More Topology and the Weil Conjectures: We will present a proof of the Lefschetz Fixed Point Theorem, and begin working on the Weil conjectures. Length will depend on if we decide to do the Riemann Hypothesis or not. If not, then we should be able to fit the material into $1 + \epsilon$ lectures.

Lecture n+1

Special Topics I: Etale Homotopy Types. We will gauge the audience's interest and see what people want to discuss, but Carlos said he is interested in etale homotopy types. We could always have more topics later as well (time-permitting).

Sources

1. Milne's *Etale Cohomology* Notes

2. Milne's *Etale Cohomology* Book (unclear how different these two sources are)

3. Freitag & Kiehl's *Etale Cohomology and the Weil Conjecture*

4. Course Notes for a course given by Daniel Litt on this exact topic (notes can be found on his website, and his lectures were recorded on YouTube)