LEARNING SEMINAR ON MANIFOLDS AND HOMOTOPY THEORY

ABSTRACT. As the goal of life of homotopy theorists is to compute the homotopy groups of spheres, geometric topologists on the other hand dream to compute the cohomology groups of the moduli of surfaces $\mathcal{M}_g$. Most of what we know comes from the following idea. Letting g goes to ∞, we first compute the stable cohomology $H^*(\mathcal{M}_\infty)$. Secondly, the range in which $H^*(\mathcal{M}_g, \mathcal{M}_{g-1})$ vanishes is an affine function of g. This is what we call homological stability. In this seminar, we plan on covering the cellular $\mathbb{E}_k$-algebras techniques to prove homological stability. We also plan to discuss applications to the cohomology of general linear groups of fields, creating a connection with algebraic K-theory. In the last section of the seminar, we will discuss even more surprising connections between manifold theory and algebraic K-theory and other homotopy-theoretic objects. Depending on the participants' interests, we may also discuss applications to arithmetic statistics or representation theory.

FIGURE 1. not the best picture but TBD

Topic Ideas:
This is a non-exhaustive list of topics we could cover:

(1) Introduction: kinda announce main results and characters we want to discuss in this seminar
(2) What is M_g? From AG to AT
(3) Stable homology of moduli spaces of surfaces
(4) Proof of Dundas-McCarthy Theorem and Goodwillie Calculus
(5) Homological Stability and algebraic K-theory
(6) Power Operations on E_k-algebras
(7) Stable h-cobordisms and K-theory
(8) Dwyer-Weiss-Williams stuff (always wanted to read this, something with K theory and manifolds)
(9) Some spectral sequences
(10) (G)-stuff: unstable equivariant homotopy, genuine, naive

1. Introduction

Introduce topics we wanna discuss in seminar and how they should connect together.

2. All you need to know about $\mathcal{M}_g$

2.1. **Curves, surfaces, mapping class groups.**

2.2. **Stable homology of $\mathcal{M}_g$.**

2.3. **Homological Stability for mapping class groups.**

3. A general method to prove homological stability

3.1. **What are $\mathbb{E}_k$-algebras?**

3.2.

3.3.

3.4. **Power Operations.**

3.5. **Examples in Number Theory.**

4. General linear groups and Algebraic K-theory

4.1.

4.2.

4.3.

5. More on Algebraic K-theory

5.1. **Another type of stability result: the parametrized h-cobordism theorem.**

5.2. **Dwyer-Weiss Williams paper.**

5.3. **The Dundas-McCarthy Theorem and its proof.**